

Extracting Hadron Parameters from Dispersive Sum Rules

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Puzzled or surprised by the almost incredible accuracy occasionally claimed in the literature to be achievable for numerical outcomes of QCD sum-rule analyses, we scrutinized the usual procedure employed for the extraction of the parameters of individual bound states from dispersive sum rules by taking advantage of the exact solvability of a quantum-mechanical harmonic-oscillator model: It turns out that the *determination of the ground-state parameters* (that is, decay constant and form factor) *by requiring independence from the Borel mass in its stability window does not necessarily yield their exact numerical values* [1 – 6]. For instance, the comparison of the sum-rule predictions for bound-state parameters with their numerical values known precisely in our harmonic-oscillator model reveals that standard sum-rule procedures underestimate the ground-state decay constant by some 4% and its form factor by almost 15%; such systematic uncertainties cannot be inferred from our correlators' accuracy better than 1% in the window of Borel stability: they are uncontrollable.

8th Conference Quark Confinement and the Hadron Spectrum
September 1–6, 2008
Mainz, Germany

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The idea behind this sequence of studies at quantum-physics level [1 – 6] is simple and elegant: For a harmonic-oscillator model defined by the nonrelativistic Hamiltonian $H = \mathbf{p}^2/2m + m\omega^2\mathbf{x}^2/2$ correlators, such as the polarization $\Pi(E) \equiv \langle \mathbf{x}_f = \mathbf{0} | (H - E)^{-1} | \mathbf{x}_i = \mathbf{0} \rangle$, are known at both “hadron” and “quark” levels, where the counterpart of the operator product expansion in QCD is easily found; at “hadron” level, the Borel transform of $\Pi(E)$, with Borel mass μ , is $\Pi(\mu) = \sum_{n=0}^{\infty} R_n \exp(-E_n/\mu)$ with (exact) energies $E_0 = 3\omega/2$, $E_1 = 7\omega/2$, \dots and decay constants $R_n \equiv |\Psi_n(\mathbf{x} = \mathbf{0})|^2$. To extract from the quark level the ground-state parameters E_0 and R_0 numerically, *duality*, i.e., equality of the continuum contributions above a threshold energy is assumed. Equating the remainders of “hadron” and “quark” expressions yields a *sum rule*. This is best illustrated graphically (Fig. 1). However, the threshold’s μ -dependence induces uncontrolled uncertainties into this approach; without specifying this dependence sum-rule predictions are not reliable but plagued by considerable systematic errors.

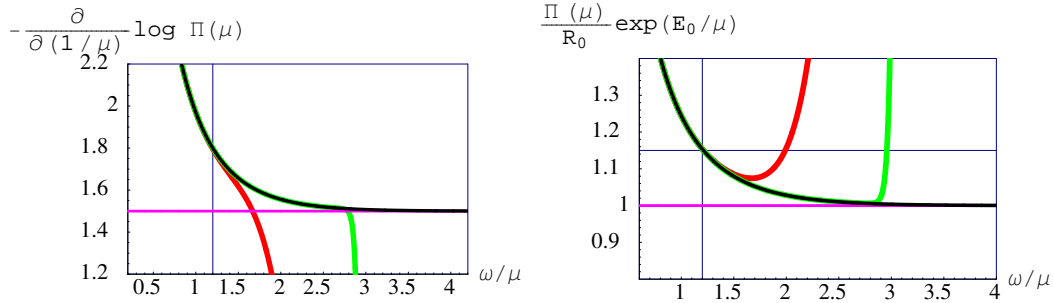


Figure 1: Extraction of the ground-state energy E_0 and decay constant R_0 of our model by either considering the exact expression (black) for $\Pi(\mu)$, or retaining 4 (red) and 100 (green) terms in its expansion for large μ .

Acknowledgements. D. M. gratefully acknowledges financial support from the Austrian Science Fund (FWF) under projects P17692 and P20573, and from the RFBR under project 07-02-00551.

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